EPRS8550
Matrix Operations

Why do we need matrix operations?

Example: Slopes and Intercept

When k = 1

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$b = r \frac{s_y}{s_x}$$

When k = 2

$$b_0 = \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2$$

$$b_1 = \frac{r_{y1} - r_{y2} \cdot r_{12}}{1 - r_{12}^2} \cdot \frac{s_y}{s_{x_1}}$$

$$b_2 = \frac{r_{y2} - r_{y1} \cdot r_{12}}{1 - r_{12}^2} \cdot \frac{s_y}{s_{x_2}}$$

When k = 3?

When k = 15?

In Matrix operations:

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

Regression Analysis: Matrix Operations

$$Y_i = b_0 + b_1 X_{i1} + b_2 X_{i2} + \ldots + b_k X_{ik} + e_i$$

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ . \\ . \\ . \\ Y_N \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1k} \\ 1 & X_{21} & X_{22} & \cdots & X_{2k} \\ 1 & X_{31} & X_{32} & \cdots & X_{3k} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ 1 & X_{N1} & X_{N2} & \cdots & X_{Nk} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ . \\ . \\ . \\ b_k \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ . \\ . \\ . \\ e_N \end{bmatrix}$$

Quick Review of Basic Matrix Algebra

1. Addition $\mathbf{A}+\mathbf{B}=\mathbf{C}$

$$\begin{bmatrix} 6 & 4 \\ 5 & 6 \\ 9 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 4 \\ 7 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 12 & 10 \\ 10 & 8 \end{bmatrix}$$

2. Subtraction $\mathbf{A}-\mathbf{B}=\mathbf{C}$

$$\begin{bmatrix} 6 & 4 \\ 5 & 6 \\ 9 & 5 \end{bmatrix} - \begin{bmatrix} 7 & 4 \\ 7 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -2 & 2 \\ 8 & 2 \end{bmatrix}$$

3. Multiplication $\mathbf{AB}=\mathbf{C}$

$$\begin{bmatrix} 3 & 1 \\ 5 & 1 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 1 & 4 \\ 5 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 9 & 14 \\ 25 & 11 & 22 \\ 28 & 26 & 16 \end{bmatrix}$$

3 X 2 2 X 3 3 X 3

4. Inverse $\mathbf{B}=\mathbf{A}^{-1}\mathbf{C}$ (in ordinary algebra, $b=\frac{1}{a}\cdot c$)

Given **A** and **B**, two square matrices, if **AB = I**, then **A** is the inverse of **B**.

5. Transpose

$$\mathbf{A}=\begin{bmatrix} \mathbf{6} & \mathbf{4} \\ \mathbf{5} & \mathbf{6} \\ \mathbf{9} & \mathbf{5} \end{bmatrix} \qquad \mathbf{A}'=\begin{bmatrix} \mathbf{6} & \mathbf{5} & \mathbf{9} \\ \mathbf{4} & \mathbf{6} & \mathbf{5} \end{bmatrix}$$

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

$$\mathbf{X}'\mathbf{X} = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&2&2&2&2&3&3&3&3&4&4&4&4&5&5&5&5 \end{bmatrix} \begin{bmatrix} 1&1 \\ 1&1 \\ 1&1 \\ 1&1 \\ 1&2 \\ 1&2 \\ 1&2 \\ 1&2 \\ 1&3 \\ 1&3 \\ 1&3 \\ 1&3 \\ 1&4 \\ 1&4 \\ 1&4 \\ 1&4 \\ 1&5 \\ 1&5 \\ 1&5 \\ 1&5 \end{bmatrix} = \begin{bmatrix} 20 & 60 \\ 60 & 220 \end{bmatrix} = \begin{bmatrix} N & \sum X \\ \sum X & \sum X^2 \end{bmatrix}$$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} .275 & -.075 \\ -.075 & .025 \end{bmatrix} \qquad \mathbf{X}'\mathbf{y} = \begin{bmatrix} \mathbf{146} \\ \mathbf{468} \end{bmatrix} = \begin{bmatrix} \sum Y \\ \sum XY \end{bmatrix}$$

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} = \begin{bmatrix} .275 & -.075 \\ -.075 & .025 \end{bmatrix} \begin{bmatrix} 146 \\ 468 \end{bmatrix} = \begin{bmatrix} 5.05 \\ .75 \end{bmatrix}$$