

One Qualitative Independent Variable (ANOVA)

t-test Revisited

Recall t-test

Group 1 Group 2

10	9
15	6
13	11
12	5
9	7

\bar{Y}_j	11.8	7.6
S_y^2	5.7	5.8
S_y	2.387	2.408

$$H_0: \mu_1 = \mu_2$$

$$t = \frac{Y_1 - Y_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{11.8 - 7.6}{\sqrt{\frac{5.7}{5} + \frac{5.8}{5}}} = \frac{4.2}{1.516} = 2.77$$

$$t^* = t^*_{\alpha/2, n_1+n_2-2} = t^*_{0.5/2, 8} = 2.306$$

Therefore, reject H_0 .

Regression Analysis

Let Z = group membership

If Group 1, let Z = 1, Group 2, let Z = 0.

$$Y' = b_0 + b_1Z$$

Y Z

10	1
15	1
13	1
12	1
9	1
9	0
6	0
11	0
5	0
7	0

$$b_1 = \frac{\sum (Z - \bar{Z})(Y - \bar{Y})}{\sum (Z - \bar{Z})^2} = \frac{10.5}{2.5} = 4.2$$

$$b_0 = \bar{Y} - b_1\bar{X} = 9.7 - 4.2(.5) = 7.6$$

$$Y' = 7.6 + 4.2Z$$

7.6 is the mean of Group 2

4.2 is the difference of two means

ANOVA Table				
Source	SS	df	MS	F
Reg	44.1	1	44.1	7.67***
Res	46	8	5.75	
Tot	90.1	9		

$$F^* = F_{0.5, 1, 8} = 5.32$$

Therefore, reject H_0 .

$$\text{Note: } 2.77 * 2.77 = 7.67$$

$$2.306 * 2.306 = 5.32$$

M 9.7 .5

$$H_0: \beta_1 = 0$$

Alternatively, you can test the slope.

$$t = \frac{b_1}{S_{b_1}} = \frac{4.2}{1.517} = 2.77$$

$$t^* = t^*_{0.5/2, 8} = 2.306$$

Test of slope = test of means