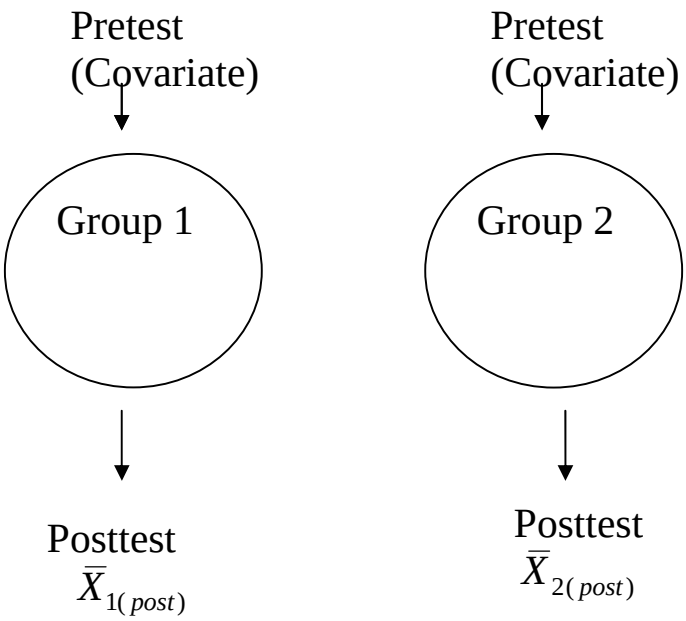
Analysis of Covariance

Design



Example

Cov: IQ IV: Groups (TRT1, TRT2)
DV: Science Achievement

$R_{IQ, Ach} = .8$ $r^2 = .64$

t-test $t(22) = 1.676, p > .05$
ANCOVA $F(1,21) = 2.25^2 = 5.06, p < .05$

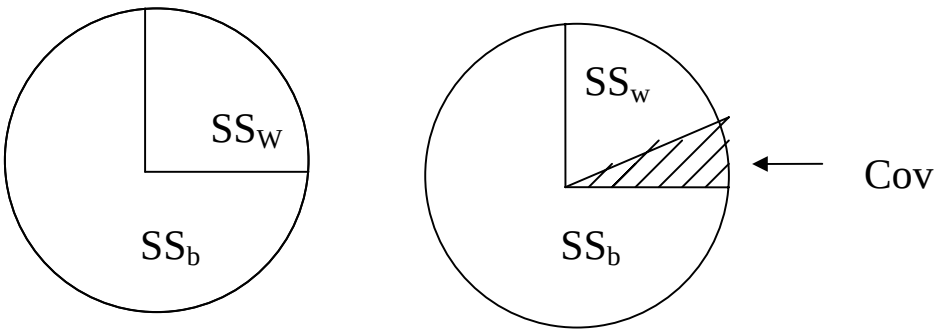
Purposes of ANCOVA

1. Adjust initial differences.

IQ:	120.2	105.8
Group	Method 1	Method 2
Posttest	73.4	67.5

2. More power.

$$F = \frac{MS_b}{MS_w} \qquad \text{(Smaller } MS_w\text{)}$$

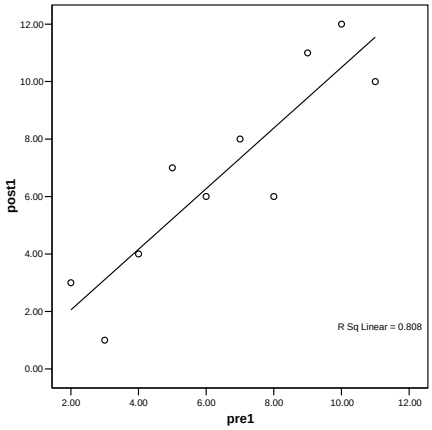


Choice of Covariate(s) – Can have multiple Cov's.
Each Cov – high r with posttest
Among Cov's – Low r

Assumption

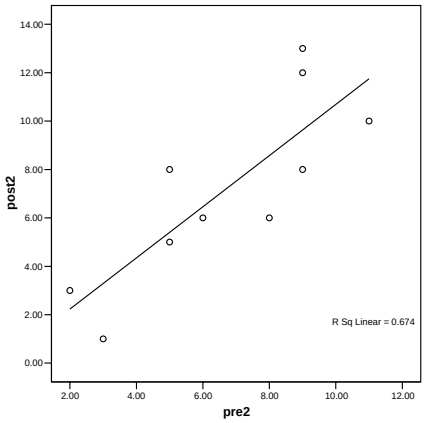
Equal population regression slopes for all groups.

Group 1



$$Y_1 = a_1 + b_1 X_1$$

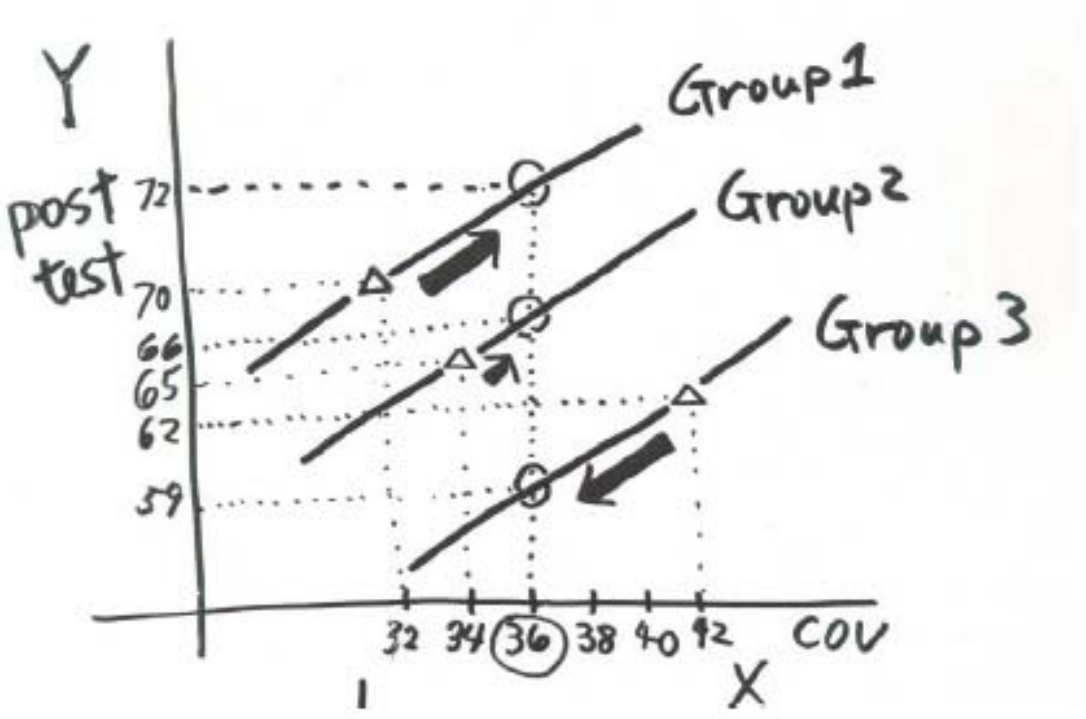
Group 2



$$Y_2 = a_2 + b_2 X_2$$

Assume $\beta_1 = \beta_2$

Adjustment on posttest



	Group 1	Group 2	Group 3
COV mean $\bar{X}_i$	32	34	42
Unadjusted Posttest Mean $\bar{Y}_i$	70	65	62
Adjusted Posttest Mean $\bar{Y}_i^*$			

$\bar{Y}_i^* = \bar{Y}_i - b(\bar{X}_i - \bar{X})$ where b = .5

$\bar{Y}_1^* = 70 - .5(32 - 36) = 72$

$\bar{Y}_2^* = 65 - .5(34 - 36) = 66$

$\bar{Y}_3^* = 62 - .5(42 - 36) = 59$

Numerical Example

Group 1 (Computer)		Group 2 (Lectures)		Group 3 (Combined)	
x	y	x	y	x	y
12	26	11	32	6	23
10	22	12	31	13	35
7	20	6	20	15	44
14	34	18	41	15	41
12	28	10	29	7	28
11	26	11	31	9	30
Means					
11	26	11.3	30.67	10.83	33.50

Correlations

.97 .98 .93

$r_{xy} = .85$

Suppose we did one-way ANOVA on the posttest scores.

Sourse	SS	df	MS	F
GP	172.11	2	86.06	1.94 NS
Error	666.83	15	44.96	

$F_{.05,2,15} = 3.68$

ANCOVA

1. Test Equal Within-Group Regression Slope

Source	SS	df	MS	F	
COV(X)	503.69	1	503.69	139.47	
GP	10.13	2	5.06	1.40	
COV*GP	2.55	2	1.28	.35	$H_0: \beta_1 = \beta_2 = \beta_3$ (fail to reject)
Error	43.44	12	3.61		

2. ANCOVA

Source	SS	df	MS	F	
COV(X)	620.94	1	620.94	189.43***	$H_0: \beta = 0$
GP	182.82	2	91.41	27.89***	$H_0: \mu_{1adj} = \mu_{1adj} = \mu_{1adj}$
Error	45.89	14	3.28		

→

3. Calculate adjusted means $\bar{Y}^*$ and do Bryant-Paulson Post Hoc Procedure.

$$BP = \frac{\bar{Y}_1^* - \bar{Y}_2^*}{\sqrt{MSw^* \left[1 + \frac{MSbx}{MSwx}\right] / n}} \qquad \text{Note } \frac{MSbx}{MSwx} = F$$

1 vs. 2

$$BP = \frac{26.10 - 30.16}{\sqrt{3.278 \left[1 + \frac{.389}{12.27}\right] / 6}} = \frac{-4.06}{\sqrt{.56}} = -5.41^*$$

1 vs. 3

$$BP = \frac{26.10 - 33.91}{\sqrt{3.278 \left[1 + \frac{.389}{12.27}\right] / 6}} = \frac{-4.06}{\sqrt{.56}} = -10.36^*$$

2 vs. 3

$$BP = \frac{30.16 - 33.91}{\sqrt{3.278 \left[1 + \frac{.389}{12.27}\right] / 6}} = \frac{-3.75}{\sqrt{.56}} = -5.01^*$$

CV = 3.85 (df for error =14, COV = 1, Group = 3)

Assumptions of ANCOVA

ANOVA assumptions plus

1. Linearity between x and y.
2. Homogeneity of regression slopes.
3. Covarites are measured without error.

Violations

1. Curvilinear --- Adjustment of means will be improper.
2. Serious – Use Johnson-Neyman Technique.
3. Random design --- Decreased power.
   Non-random design – Can be seriously biased.

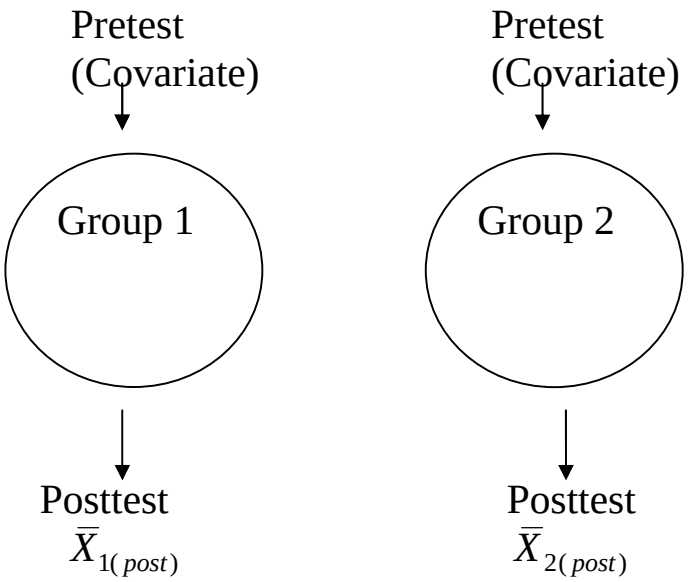
Use of ANCOVA with Intact Groups

Dangers!

1. Not equate intact groups --- Other variables unaccounted for
2. Groups that are equal on the covariate might not exist in the real world.
3. Violation of assumption more likely
4. Differential growth (AKA Selection by maturation)

Treacherous!

Alternative Analyses



If pretest and posttest are on the same scale, then

1. ANOVA on gain scores – Not reliable
2. Two-way repeated measures
   - Factor A Group
   - Factor B Time (Pre vs. Post)
   - A*B