Questions: How are the effect size measures, Cohen's f and SPSS's η^2, related? What is the difference between η^2 and partial η^2 that I see in SPSS?

Questions above are very common and students are often confused when they try to understand what η^2 and partial η^2 mean in SPSS. Please see Appendix A for a sample question and an answer from SPSS. Students usually learn Cohen's f from a textbook in the context of Power Analysis for ANOVA. Then, they find later that in SPSS η^2 and partial η^2 are the measures for effect size and they encounter a problem relating f to η^2.

Here is what they learn in the text books:

1. $\hat{f} = \sqrt{(k-1)F / N}$ (Equation 1)
2. $\eta^2 = \dfrac{f^2}{1+f^2}$ (Equation 2)
3. $\lambda = N \cdot f^2$ (Equation 3)

They also learn from SPSS that

1. $\eta^2 = \dfrac{SSH}{SST}$ (Equation 4)
2. $partial\eta^2 = \dfrac{SSH}{SSH + SSE}$ (Equation 5)

In the context of one-way ANVOA, SSH = SSB (Sum of Squares Between). In the context of factorial ANOVA (say, Factor A, Factor B, and A X B), SSH = SS(A), SS(B), or SS(AB). Since in one-way ANOVA, SSB + SSE = SST, you get the same value for η^2 and partial η^2.

Let's see an SPSS output from the toy price data (one-way ANOVA).

Tests of Between-Subjects Effects

Dependent Variable: Price

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power(a)
Corrected Model	23.200(b)	2	11.600	3.120	.093	.409	6.239	.456
Intercept	346.150	1	346.150	93.088	.000	.912	93.088	1.000
Store	23.200	2	11.600	3.120	.093	.409	6.239	.456
Error	33.467	9	3.719					
Total	398.000	12						
Corrected Total	56.667	11						

a Computed using alpha = .05
b R Squared = .409 (Adjusted R Squared = .278)

Here, η^2 = partial η^2 = SSB/SST = 23.2/56.667 = .409. So far so good.

Let's see an SPSS output from our two-way data.

Tests of Between-Subjects Effects

Dependent Variable: Score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power(a)
Corrected Model	120.000(b)	5	24.000	5.538	.007	.698	27.692	.921
Intercept	2178.000	1	2178.000	502.615	.000	.977	502.615	1.000
Sex	72.000	1	72.000	16.615	.002	.581	16.615	.962
TRT	21.000	2	10.500	2.423	.131	.288	4.846	.394
Sex * TRT	27.000	2	13.500	3.115	.081	.342	6.231	.490
Error	52.000	12	4.333					
Total	2350.000	18						
Corrected Total	172.000	17						

a Computed using alpha = .05
b R Squared = .698 (Adjusted R Squared = .572)

Here, partial η^2 for Sex = SSA/(SSA+SSE) = 72/(72+52) = .581, partial η^2 for TRT= SSB/(SSB+SSE) = 21/(21+52) = .288, partial η^2 for Sex*TRT = SSAB/(SSAB+SSE) = 27/(27+52) = .342. So far so good.

Now, let's look at one-way ANOVA table from SPSS above. Noncent. Parameter is λ (6.239). From Equation 3, you may be tempted to obtain $f^2 = \frac{\lambda}{N} = \frac{6.239}{12} = .52$ (so $f = \sqrt{.52} = .72$) or from Equation 1, $\hat{f} = \sqrt{(k-1)F/N} = \sqrt{(3-1)3.12/12} = .72$. Now, if you use this f, then from Equation 2,

$\eta^2 = \frac{f^2}{1+f^2} = \frac{.52}{1+.52} = .342$. However, in SPSS, η^2 is reported as .409. This is where students get confused.

The key here is what David Nichols says in Appendix A. In SPSS, $\lambda = df_e \cdot f^2$, not $\lambda = N \cdot f^2$. So, $\lambda = df_e \cdot f^2 = 9 \cdot f^2$. Then, $f^2 = .693$. $f = .832$. Plugging in this f^2 in Equation 2, you get $\eta^2 = \frac{f^2}{1+f^2} = \frac{.693}{1+.693} = .409$.

So, there are two kinds of f^2. Let's call (just in this paper), f obtained from Equation 1 is f_c (.72) and f obtained from SPSS is f_s (.83). Then, the relationship between f_c and f_s is

$$f_s^2 = \frac{N \cdot f_c^2}{df_e} \quad . \qquad \text{(Equation 6)}$$

So Equation 2 can be written as

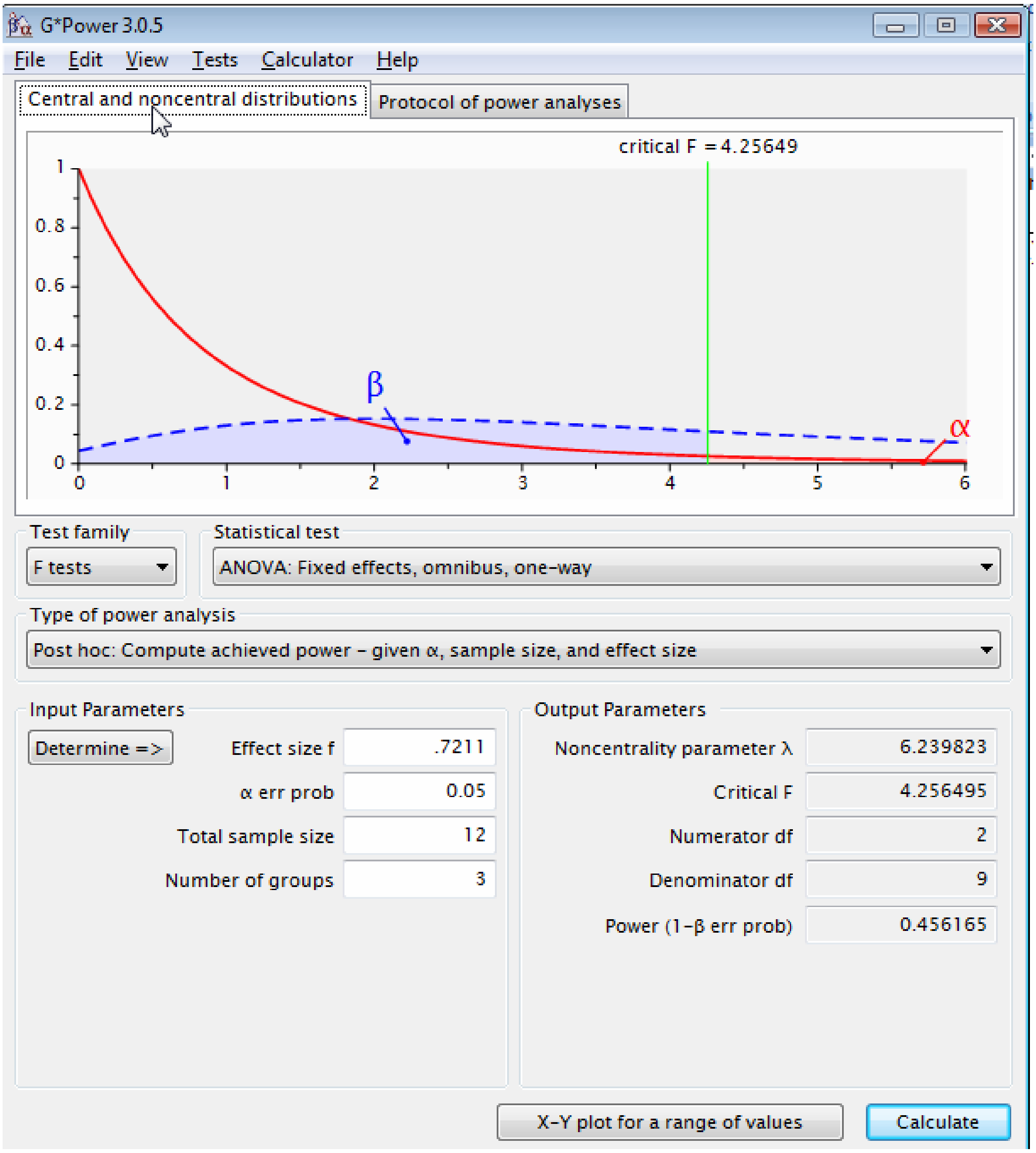
$$\eta^2 = \frac{f_s^2}{1+f_s^2} = \frac{\frac{N \cdot f_c^2}{df_e}}{1+\frac{N \cdot f_c^2}{df_e}} \qquad \text{(Equation 7)}$$

The equation became a little heavy, but now you see the relationship between Cohen's f and SPSS's η.

Suppose you get Cohen's $\hat{f}$ from Equation 1. That is,
$\hat{f} = \sqrt{(k-1)F/N} = \sqrt{(3-1)3.12/12} = .72$. Plugging this .72 in Equation 7,

$$\eta^2 = \frac{f_s^2}{1+f_s^2} = \frac{\frac{N \cdot f_c^2}{df_e}}{1+\frac{N \cdot f_c^2}{df_e}} = \frac{\frac{12 \cdot .52}{9}}{1+\frac{12 \cdot .52}{9}} = \frac{.69}{1+.69} = .409$$

Now, G*Power uses Cohen's f.



Here λ is 6.24, and power is.456. These values are the same as you obtained from SPSS.

Appendix A

SPSSX-L archives -- June 2000 (#266)

Date: Fri, 23 Jun 2000 16:24:48 -0500

Reply-To: "Nichols, David" <nichols@SPSS.COM>

Sender: "SPSSX(r) Discussion" <SPSSX-L@LISTSERV.UGA.EDU>

From: http://listserv.uga.edu/cgi-bin/wa?A2=ind0006&L=spssx-l&D=0&P=31356

Subject: Re: Better definition of eta squared problem

Comments: To: "Perkins, Nigel" <N.R.Perkins@MASSEY.AC.NZ>

If you look at the SPSS algorithms Cohen's book and do some algebra, you can find that SPSS is obtaining the noncentrality parameter lambda value as DFE*(SSH/SSE), while Cohen is obtaining it as N*(SSH/SSE). The difference presumably comes from the fact that Cohen is treating the whole situation as one in which everything is known, while SPSS is estimating things.

David Nichols
Principal Support Statistician and
Manager of Statistical Support
SPSS Inc.

> -----Original Message-----

> From: Perkins, Nigel [mailto:N.R.Perkins@MASSEY.AC.NZ]

> Sent: Sunday, June 11, 2000 6:04 PM

> To: SPSSX-L@LISTSERV.UGA.EDU

> Subject: [SPSSX-L] Better definition of eta squared problem

>
>
> Dear All,
> Thanks for your replies. I guess I did not explain my problem
> clearly enough
> so here goes again.
> The problem surfaced in a 2way ANOVA but after wondering
> whether partial
> eta^2 might be playing a role, I went back to a 1-way ANOVA
> to try and use a
> simple example to make things work out.
> My goal was to understand the relationship between the SPSS
> eta^2 term and
> other power related parameters. I can see how the eta^2 term
> in SPSS is
> produced (SSB/SST) but I am having trouble relating this to
> the estimates of
> lambda and power. What I set out to try and do was try and
> use relatively
> simple data sets and to estimate f, lambda, power, eta^2 etc
> using SPSS,
> PASS, raw data and text derived formulae etc to deepen my
> understanding of
> the parameters and how they are related. Everything seemed to be going
> rosily until I tried to use the SPSS eta^2 term to generate
> estimates of
> lambda and f^2 by using formulae derived from texts such as
> cohen and kirk.

>

> Cohen and Kirk and others mention that eta^2 is related to

> effect size (f^2)

> in the following manner:

>   f^2 = eta^2 / (1-eta^2)

> Then

>   lambda (noncentrality parameter) = f^2*N

>   where N is the total sample size

>

> In addition, f^2 = SD(means) / SD(population)

>

>   where SD(means) = SUM(ui-u)^2/k (k=number of levels in a 1-way

> ANOVA; ui = mean of the ith group and u = grand mean).

>   SD(population) = MSerror term from ANOVA

>

>

> Now, my problem is this:

>   Using the formulae for deriving f^2 from variance

> estimates and then

> converting it to lambda and then using the noncentral

> F-distribution, I can

> produce estimates of power and lambda which are exactly the

> same as those

> which I can produce using the options buttons in SPSS or using power

> programs like PASS.

>   BUT, when I attempt to use the SPSS eta^2 term to produce an

> estimate of f^2 (using f^2=eta^2/(1-eta^2) and then using the

> f^2 term to

> derive lambda and power, the estimates do not match up with any of the
> estimates I get by deriving f^2 and lambda directly from the data.
> Nigel
>
>
>
>
> -----Original Message-----
> From: Nichols, David [mailto:nichols@SPSS.COM]
> Sent: Saturday, June 10, 2000 3:54 AM
> To: SPSSX-L@LISTSERV.UGA.EDU
> Subject: Re: eta squared in UNIVARIATE GLM
>
>
> The eta^2 in GLM/UNIANOVA is indeed a partial eta^2, but since Nigel
> indicates that he's not matching results even in a one way
> situation, that's
> not the entire issue. Since he didn't say what formula he's
> using, I'm not
> sure why the results aren't matching what SPSS does. For a
> one way ANOVA,
> the partial eta^2 is the same as the total eta^2, just
> SSB/SST, where SSB is
> the between groups sum of squares and SST is the mean
> corrected total sum of
> squares.
>
> By the way, MANOVA also gives only the partial eta^2 using the default

> UNIQUE sums of squares. It will only give the total eta^2 and
> omega^2 when
> using SEQUENTIAL sums of squares.
>
> On the issue of partial vs. total eta^2, note that in two way and more
> complicated designs, Cohen actually also uses the partial version.
>
> David Nichols
> Principal Support Statistician and
> Manager of Statistical Support
> SPSS Inc.
>
> > -----Original Message-----
> > From: Dale Glaser [mailto:glaser@pacific-science.com]
> > Sent: Friday, June 09, 2000 10:51 AM
> > To: SPSSX-L@LISTSERV.UGA.EDU
> > Subject: Re: [SPSSX-L] eta squared in UNIVARIATE GLM
> >
> >
> > If I recall, technically it is a partial eta-squared, as
> > opposed to eta
> > squared; if you use the MANOVA option (via syntax only), it
> > provides both
> > eta squaredand partial eta squared, whereas with the GLM
> > option only the
> > partial is reported even though it is mistakenly labeled as
> > eta squared
> > (Dave Nichols please correct me if I'm off base

> > here)..........dale glaser
> >
> > -----Original Message-----
> > From: SPSSX(r) Discussion [mailto:SPSSX-L@LISTSERV.UGA.EDU]
> > On Behalf Of
> > Perkins, Nigel
> > Sent: Thursday, June 08, 2000 9:01 PM
> > To: SPSSX-L@LISTSERV.UGA.EDU
> > Subject: eta squared in UNIVARIATE GLM
> >
> > I am trying to come to grips with the eta squared term in the
> > UNIVARIATE GLM
> > procedure of SPSS (using version 9.0). I am using one way and 2-way
> > factorial ANOVAs to produce power and effect size estimates.
> >
> > Several textbooks mention a term labelled eta squared or
> > R-squared which
> > appears to be interpreted in the same way as the eta squared
> > value in SPSS
> > (percent variance explained). Textbooks such as Cohen
> > (Statistical power
> > analysis for the behavioral sciences) and Kirk (Experimental
> > design) mention
> > that eta squared should be related to effect size in the
> > following manner:
> >
> > f-squared = eta squared / (1- eta squared)
> >

> > However, when I attempt to use a text book derived formula

> > for estimating

> > eta squared and then compare it to the SPSS value, they are

> > not the same!

> > This occurs even in a 1-way ANOVA (using a simple example)

> > when degrees of

> > freedom estimation is straight forward and not term dependent.

> >

> > Can anyone explain why the SPSS eta-squared term does not

> appear to be

> > related to the effect size term in the way that Cohen and

> > Kirk and others

> > say that it should be?

> > Regards,

> > Nigel Perkins, BVSc, MS, Dip ACT, FACVSc

> > Senior Lecturer, Veterinary Epidemiology

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> >

>

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