Regression Equation for Prediction

Instructor: "The average grade in this class is B, but some students do better and other students do not do so well. Also, how much time students spend studying is related to their test performance."

Question 1: "How well do students do who study, say, 4 hours for the exam?"

--- Predict the average Y for a given X ( $\mu_{Y/X_0}$ )

Question 2: "If I study for 4 hours, how well can I expect to do?"

--- Predict a value for Y for a given X ( $Y'_{X_0}$ )

Recall Y' = 12.79 + 1.47X

If X = 4, Y' = 12.79 + 1.47(4) = 18.67

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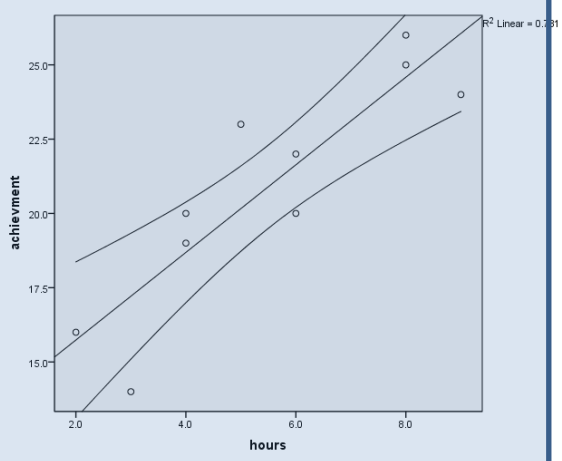
Predicted value for

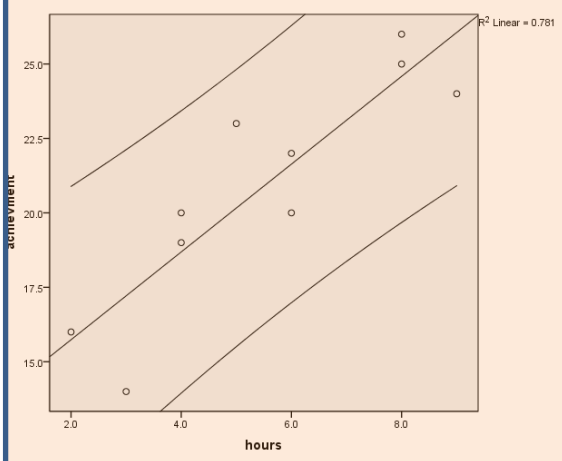
| | Conditional mean (Question 1) | Individual (Question 2) |
|---|---|---|
| Y' | 18.67 | 18.67 |
| SE | $S_{\mu_{Y/X_0}} = S_{Y.X}\sqrt{\frac{1}{N}+\frac{(X_0-\bar{X})^2}{(N-1)S_X^2}}$ | $S_{Y'} = S_{Y.X}\sqrt{1+\frac{1}{N}+\frac{(X_0-\bar{X})^2}{(N-1)S_X^2}}$ |
| | Smaller | Larger (due to ind. difference) |

Note that as $X_0$ departs from $\bar{X}$ , the larger the SE

| $X_0$ | Y′ | $S_{\mu_{Y/X_0}}$ | $S_{Y'}$ |
|---|---|---|---|
| 1 | 14.26 | 1.38 | 2.366 |
| 3 | 17.21 | .92 | 2.129 |
| 5 | 20.14 | .62 | 2.018 |
| 7 | 23.11 | .74 | 2.056 |
| 9 | 26.06 | 1.47 | 2.233 |




95% Confidence Interval for Mean




95% confidence Interval for Individual

Interval Estiation for $X_0 = 4$

$$Y' \pm t_{\alpha/2,N-k-1} \cdot (SE)$$

Conditional Mean

$$S_{\mu_{Y/X_0}} = S_{Y.X}\sqrt{\frac{1}{N} + \frac{(X_0 - \bar{X})^2}{(N-1)S_X^2}}$$

$$= 1.92\sqrt{\frac{1}{10} + \frac{(4-5.5)^2}{(10-1)5.38}} = .735$$

$$18.7 \pm 2.306(.735)$$

$$= 18.7 \pm 1.69$$

(17.01, 20.39)

Individual

$$S_{Y'} = S_{Y.X}\sqrt{1 + \frac{1}{N} + \frac{(X_0 - \bar{X})^2}{(N-1)S_X^2}}$$

$$= 1.92\sqrt{1 + \frac{1}{10} + \frac{(4-5.5)^2}{(10-1)5.38}} = 2.055$$

$$18.7 \pm 2.306(2.055)$$

$$= 18.7 \pm 4.74$$

(13.96, 23.44)

Hypothesis Testing for $X_0 = 4$ (Suppose 16 is the cutoff for a B)

$H_0$: $\mu_{Y/X_0} = 16$

$$t = \frac{Stat - Par}{SE} = \frac{18.68 - 16}{.735} = 3.659$$

$$t^*_{\alpha/2,10-1-1} = 2.306$$

Reject $H_0$

$H_0$: population $Y'_{X_0} = 16$

$$t = \frac{Stat - Par}{SE} = \frac{18.68 - 16}{2.055} = 1.304$$

Fail to reject $H_0$