Partial and Semipartial Correlation

Review

R-square： $R^2 = \frac{SSreg}{SStot} = r^2_{y\hat{y}}$ In our data R^2=.82

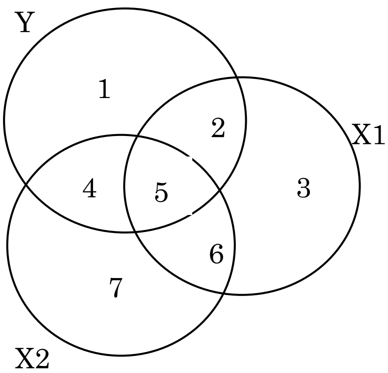
Multiple R： $\sqrt{R^2} = r_{y\hat{y}}$ In our data R=.91

$$R^2_{y.12} = \frac{r^2_{y1} + r^2_{y2} - 2r_{y1} \cdot r_{y2} \cdot r_{12}}{1 - r^2_{12}}$$

Partial Correlation Coefficient

A measure of the strength of the linear relationship between two variables after controlling for the effects of other variables

Venn diagram



$$r_{y1.2}^2 = \frac{2}{1+2}$$

$\hat{Y} = a + bX_2$ $\quad e_1 = Y - \hat{Y}$

$\hat{X}_1 = c + dX_2$ $\quad e_2 = X_1 - \hat{X}_1$ $\quad r_{e_1 e_2} = r_{y1.2}$

$$r_{y1.2} = \frac{r_{y1} - r_{y2} \cdot r_{12}}{\sqrt{1 - r_{y2}^2} \cdot \sqrt{1 - r_{12}^2}}$$

For our data

$r_{y1} = .8840$

$r_{y2} = .8745$

$r_{12} = .8826$

The r between hours of study and test scores <u>independent of student motivation</u> (<u>with motivation held constant</u> or <u>motivation partialed out</u>) is

$$r_{y1.2} = \frac{.8840 - .8745 \cdot .8826}{\sqrt{1-(.8745)^2} \cdot \sqrt{1-(.8826)^2}} \approx .49$$

Similarly, the r between motivation and test scores independent of hours of study is

$$r_{y2.1} = \frac{.8745 - .8840 \cdot .8826}{\sqrt{1-(.8840)^2} \cdot \sqrt{1-(.8826)^2}} \approx .43$$

Partial Correlation via R^2

$$r_{y1.2}^2 = \frac{R_{y.12}^2 - R_{y2}^2}{1 - R_{y2}^2} = \frac{.822 - .8745^2}{1 - .8745^2} = .24$$

$$r_{y1.2} = \sqrt{.24} \approx .49$$

$$r_{y2.1}^2 = \frac{R_{y.12}^2 - R_{y1}^2}{1 - R_{y1}^2} = \frac{.822 - .8840^2}{1 - .8840^2} = .185$$

$$r_{y2.1} = \sqrt{.185} \approx .43$$

Extension to 3X's

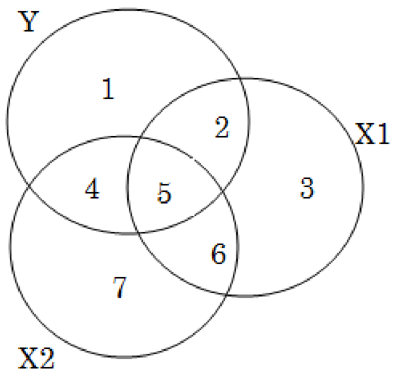
$$r_{y1.23}^2 = \frac{R_{y.123}^2 - R_{y.23}^2}{1 - R_{y.23}^2}$$

Extension to k X's

$$r_{y1.2\cdots k}^2 = \frac{R_{y.12\cdots k}^2 - R_{y.2\cdots k}^2}{1 - R_{y.2\cdots k}^2}$$

Semipartial or Part Correlation

Venn diagram



$$r^2_{y1.2(s)} = \frac{2}{1+2+4+5}$$

The semipartial correlation is the correlation between all of Y and that part of X1 which is independent of X2.

$$r_{y1.2(s)} = r_{y.e2}$$

Formula for Semipartial Correlation

$$r_{y1.2(s)} = \frac{r_{y1} - r_{y2} \cdot r_{12}}{\sqrt{1 - r^2_{12}}} = .24$$

Note: $r_{y2.1(s)} = .20$

or

$$r^2_{y1.2(s)} = R^2_{y.12} - R^2_{y2}$$

Squared semipartial correlation --- Unique contribution of that variable <u>above and beyond</u> other variables

Test of Significance for Partial and Semipartial Correlation (Also Squared Semipartial Correlation)

Our data

$$\hat{Y} = 8.85 + .846X_1 + .409X_2$$

1. Test of b $\quad t = 1.49 \quad 1.49^2 = 2.22$

Same as

2. Test of reduced model

$$F = \frac{R^2_{FM} - R^2_{RM} \big/ (K_{FM} - K_{RM})}{(1 - R^2_{FM}) \big/ (N - K_{FM} - 1)}$$

F = 2.22

Same as

3. Test of squared semipartial correlation

$r^2_{y1.2(s)}$ F = 2.22

Same as

4. Test of partial correlation

$r_{y1.2}$ t = 1.49