

Single Group Repeated Measures

Numerical Example

Table 1

Subj.	TRT			Means
	1	2	3	
1	30	28	34	30.677
2	14	18	22	18.000
3	24	20	30	24.667
4	38	34	44	38.667
5	26	28	30	28.000
Col. Means	26.4	25.6	32	28.000
Col. Var.	76.8	42.8	64	

If you perform a one-way ANOVA

Table 2

Source	SS	df	MS	F
TRT	121.6	2	60.80	.99
Error	734.4	12	61.20	
Tot	856.0	14		

If you perform a single group repeated measures ANOVA

Table 3 (In general)

Source	SS	df	MS	F
TRT	SSt _{rt}	k-1	MSt _{rt}	MSt _{rt} /MS _{res}
Sub	SS _{sub}	n-1	MS _{sub}	MS _{sub} /MS _{res}
Residual	SS _{res}	(k-1)(n-1)	MS _{res}	
Tot	SSt _{ot}	nk-1		

Note: k within levels, n subjects

Table 4. (For our data)

Source	SS	df	MS	F
TRT	121.6	2	60.80	12.67*
Sub	696.0	4	174.01	36.25*
Residual	38.4	8	4.8	
Tot	856.0	14		

* P < .05

Post Hoc Test

Recall for a one-way ANOVA

$$|\bar{X}_i - \bar{X}_j| > q_{\alpha, k, N-k} \sqrt{MSw/n}$$

Now for a single group repeated measures ANOVA

$$|\bar{X}_i - \bar{X}_j| > q_{\alpha, k, (n-1)(k-1)} \sqrt{MSres/n}$$

$$|\bar{X}_i - \bar{X}_j| > 4.401 \sqrt{4.8/5} = 3.96$$

For the example above,
Therefore, any mean difference bigger than 3.96 is a significant difference.

Means Diff.

TRT 3	32.0	
		5.6
TRT 1	26.4	
		0.8
TRT 2	25.0	

Conclusions: The mean for TRT3 is significantly higher than the mean for TRT1 or TRT2. There is no significant difference in means for TRT1 and TRT2.

Assumptions

1. Independence of observations
2. Multivariate Normality
3. Sphericity (ϵ -- Epsilon)

If sphericity is met, then $\epsilon = 1$.

Worst possible $\epsilon = 1/(k-1)$. e.g., $k = 3$, $\epsilon = 1/2 = .5$

$\hat{\epsilon}$ (Greenhouse and Geisser, 1959)

$$\begin{aligned} \text{Adjust df } (k-1) &\rightarrow \hat{\epsilon}(k-1) \\ (k-1)(n-1) &\rightarrow \hat{\epsilon}(k-1)(n-1) \end{aligned}$$

$\tilde{\epsilon}$ (Huynh and Feldt, 1976)

$$\tilde{\epsilon} = \frac{n(k-1)\hat{\epsilon} - 2}{(k-1)[(n-1) - (k-1)\hat{\epsilon}]}$$

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Col. Var.	76.8	42.8	64	

Table 5

Sub	1-2	1-3	2-3
1	2	-4	-6
2	-4	-8	-4
3	4	-6	-4
4	4	-6	-10
5	-2	-4	-2
Col. Mean	.8	5.6	-6.4
Col. Var	13.2	2.8	12.8

One Between and One Within Factor (Split Plot Design)

Numerical Example

Design

TRT			DAYS			
X _A	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	
X _B	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	DV = verbal recall

1. Do groups differ?
 2. Days effect?
 3. Group by Days interaction?
- Also test polynomial (linear, quadratic, cubic)

Table 6

	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Means
X _A	34.250	30.875	24.500	19.125	16.875	24.325
X _B	36.250	31.625	24.875	20.250	15.250	25.650
	35.250	31.250	24.687	19.687	16.062	

Table 7 (In general)

Source	df	MS	F
GP (A)	J-1	MS _A	MS _A /MS _{E1}
Error 1	N-J	MS _{E1}	
Days (B)	k-1	MS _B	MS _B /MS _{E2}
A*B	(J-1)(k-1)	MS _{AB}	MS _{AB} /MS _{E2}
Error 2	(N-J)(k-1)	MS _{E2}	

Note: J groups, k within levels, N subjects

Table 8 (For our data)

Source	df	MS	F
GP (A)	1	5.51	.04
Error 1	14	126.05	
Days (B)	4	1006.29	166.38*
A*B	4	7.23	1.20
Error 2	56	6.05	

* p < .05