

### 7.11 BRYANT–PAULSON POST HOC PROCEDURE

Since the covariate(s) used in social science research are essentially always random, it is important that this be incorporated into any post hoc procedure following an analysis of covariance. This is *not* the case for the Tukey procedure. The Bryant–Paulson (1976) procedure was derived under the assumption that the covariate is a random variable. It is a generalization of the Tukey technique. Which particular Bryant–Paulson (BP) statistic we use to determine if a pair of adjusted means is significantly different depends on whether the study is a randomized (subjects

randomly assigned to the groups) or a non-randomized design, and on how many covariates are present.

Below we present the appropriate statistics for one covariate for both a randomized and non-randomized design. The statistics for the multiple covariate case are given in Stevens (1986). Note that if the group sizes are unequal, then the harmonic mean is used.

$$\begin{array}{cc} \textit{Randomized Design} & \textit{Non-Randomized Design} \\ \frac{\bar{y}_i^* - \bar{y}_j^*}{\sqrt{MS_w^*[1 + MS_{b_i}/MS_w]/n}} & \frac{\bar{y}_i^* - \bar{y}_j^*}{\sqrt{MS_w^*[2/n + (\bar{x}_i - \bar{x}_j)^2/SS_{cov}]/2}} \end{array}$$

where  $n$  is common group size,  $MS_w^*$  is the error term for covariance, and  $MS_{b_i}$  and  $MS_w$  are the mean between and within sums of squares from an analysis of variance on *only* the covariate.

To illustrate use of the Bryant–Paulson technique, let us apply it to the Sesame Street data analyzed in Table 7.3 to determine which pairs of sites are significantly different. Since this is a non-randomized design, the appropriate test statistic is

$$\frac{\bar{y}_i^* - \bar{y}_j^*}{\sqrt{MS_w^*[2/n + (\bar{x}_i - \bar{x}_j)^2/SS_{cov}]/2}}$$

Note that the denominator of this statistic must be computed *separately* for each paired comparison. In computing any of the **Bryant–Paulson** test statistics one also needs the results from an analysis on the groups using just the covariate(s). However, this was done in the previous Sesame Street analysis. Now, using the appropriate values from the selected SPSS MANOVA printout in Table 7.5, we show the calculations for each pair of groups. The number of subjects in sites 1 through 3 respectively are 60, 55, and 64. Since the groups sizes are approximately equal, we use the simple mean of 59.67, because it will not differ much from the harmonic and it simplifies calculations somewhat.

*Sites 1 and 2*

$$BP = \frac{29.58 - 35.73}{\sqrt{89.71[2/59.67 + (22.40 - 26.036)^2/18452.08]/2}} = -4.96$$

*Sites 1 and 3*

$$BP = \frac{29.58 - 28.94}{\sqrt{89.71[2/59.67 + (22.4 - 16.56)^2/18452.08]/2}} = .51$$

*Sites 2 and 3*

$$BP = \frac{35.73 - 28.94}{\sqrt{89.71[2/59.67 + (26.036 - 16.563)^2/18452.08]/2}} = 5.18$$

Now, referring to Table B.5 for the critical value of the .05 level of significance with one covariate, three groups, and 178 degrees of freedom, we find it is 3.37. Thus, we conclude that site 2 differs from sites 1 and 3, but sites 1 and 3 are not significantly different.