

ONE-WAY ANOVA

Example

Variety Stores	Dept. Stores	Discount Toy Stores				
x_{11}	3	x_{12}	4	x_{13}	4	x_{ij} – Price where i indicates subject (toy) and j indicates group (store).
x_{21}	6	x_{22}	7	x_{23}	5	
x_{31}	8	x_{32}	9	x_{33}	2	
		x_{42}	8	x_{43}	3	
				x_{53}	5	

$$\sum x_{i1} = 17 \quad \sum x_{i2} = 28 \quad \sum x_{i3} = 19$$

$$\bar{x}_{.1} = 5.67 \quad \bar{x}_{.2} = 7.00 \quad \bar{x}_{.3} = 3.80 \quad \bar{x}_{..} = 5.33$$

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

H_1 : At least one pair is different

Between variation

$$SS_b = \sum n_j (\bar{x}_{.j} - \bar{x}_{..})^2$$

$$= 3(5.67 - 5.33)^2 + 4(7.00 - 5.33)^2 + 5(3.80 - 5.33)^2 = 23.21$$

$$MS_b = \frac{SS_b}{df_b} = \frac{SS_b}{k-1} = \frac{23.21}{3-1} = 11.60 \quad (k = \# \text{ groups})$$

Within variation

$$SS_w = \sum_{j=1}^k \sum_{i=1}^n (x_{ij} - \bar{x}_{.j})^2$$

$$\begin{aligned} &= (3 - 5.67)^2 + (6 - 5.67)^2 + (8 - 5.67)^2 && \text{--Group 1} \\ &+ (4 - 7.00)^2 + (7 - 7.00)^2 + (9 - 7.00)^2 + (8 - 7.00)^2 && \text{--Group 2} \\ &+ (4 - 3.80)^2 + (5 - 3.80)^2 + (2 - 3.80)^2 + (3 - 3.80)^2 + (5 - 3.80)^2 && \text{--Group 3} \\ &= 33.47 \end{aligned}$$

$$MS_w = \frac{SS_w}{df_w} = \frac{SS_w}{N - k} = \frac{33.47}{12 - 3} = 3.72 \quad (N = \text{total sample size})$$

F test

$$F_{\text{calc}} = \frac{MS_b}{MS_w} = \frac{11.60}{3.72} = 3.12 \quad F_{\text{crit}} = F_{\alpha, dfb, dfw} = F_{.05, 2, 9} = 4.26$$

Decision

Fail to reject H_0 .

There was no significant price difference among the three store types ($F_{2, 9} = 3.12$, $P > .05$).

ANOVA Table

Source	SS	df	MS	F
Store (Between)	23.21	2	11.60	3.12
Error (Within)	33.47	9	3.72	
Total	56.67	11		

$$SStot = SSb + SSw$$

$$SStot = \sum_{j=1}^k \sum_{i=1}^n (x_{ij} - \bar{x}_{..})^2$$