

P and α

In journal articles, we see p's. For example, if you see something like $p < .05$, you know the null hypothesis is rejected at .05. So you declare a significant difference. Then, what is the difference between p and α ? If you are confused, you are not the only one.

Let's review the process of the hypothesis testing. You all learned that the decision about rejecting or failing to reject the null hypothesis depends on the comparison of an observed test statistic (e.g., t_{obs}) and a critical value (e.g., t_{cv} or sometimes also written as t_{crt}). We learned if $|t_{obs}| > |t_{cv}|$, then reject H_0 . Easy!

Figure 1 demonstrates one scenario and Figure 2 demonstrates another scenario. Let's assume in both scenarios the total sample sizes are the same so that we are talking about the same t distribution.

Look at Figure 1-a. Do you reject or fail to reject H_0 ? Since $t_{obs} < t_{cv}$, we fail to reject H_0 .

Look at Figure 2-a. Since $t_{obs} > t_{cv}$, we reject H_0 .

Now notice in Figures 1-a and 2-a, t_{cv} stays at the same spot. Because t_{cv} determines the rejection region (the shaded area), and in this case you set your rejection criterion (α) to be .05. Thus, the area beyond t_{cv} makes 5% of the area under the curve. That is what you decide as a researcher even before you collect data, and it is nothing to do with your data.

t_{obs} is the one which moves around on the x-axis depending on your data. It may or may not be in the rejection region depending on your data.

Now look at Figure 1-b. What is the difference in the shaded area between 1-a and 1-b? In 1-b, the shaded area is beyond the t_{obs} . And, the shaded area is the p-value! Now you see p-value moves around just as t_{obs} does. Look how small the shaded area is in Figure 2-b.

How do you get the p-value? Well, you need a computer for that. Most statistical software would give you the p-value. In fact, they do not usually give the alpha level. So you need to decide whether to reject the H_0 or not without comparing t_{obs} and t_{cv} . Can you do that? No problem! Given just Figure 1-b, you can tell $t_{obs} < t_{cv}$. Shade the outer 5% (that is α) on Figure 1-b. Then you see where t_{cv} would be. However, once you become good at this, there is really no need to draw curves. Just look at the p-value. In Figure 1-b, $p = .432$. The value .432 is larger than .05 (your predetermined α). Therefore, you fail to reject H_0 at $\alpha = .05$.

Now look at Figure 2-b. Remember, we rejected at $\alpha = .05$ as

depicted in Figure 2-a. The p-value is .032. Since the p-value is smaller than the α level, we reject H_0 at $\alpha = .05$.

Performing the hypothesis testing as in Figures 1-a and 2-a is what I call "the pre-computer age method". In other words, you do not have to calculate the p-value. (Remember that only the computer can calculate the exact p-value.) Most textbooks still use the approach in Figures 1-a and 2-a. That is fine. Because, this approach teaches students the concept of hypothesis testing.

The approach in Figures 1-b and 2-b is what I call "the computer age method". In other words, you can decide whether to reject H_0 or not simply based on the p-value. No need to look up the tables on the back of your textbook to get the critical value.

The latter approach is applicable to just about any inferential statistics. You may be working on z, t, F, or chi-square distribution. In any case, you will get the p-value. You can decide the significance by checking if p is smaller than your predetermined α or not.

Now, the confusion comes from the way p-values are reported in the journal articles. For example, if you are reporting data from Figure 1, you would say "There was no significant difference ($t(df=___) = ___, p > .05$). Why can't we report $p = .432$??? Well, we should. However, the tradition is strong. Remember in old days, it was not easy to calculate the p-value. If $p = .432$ is true, then $p > .05$ is also true.

I hope this helps somewhat for your understanding of p and α . If not, just remember:

If $p > .05$, you fail to reject at $\alpha = .05$
If $p < .05$, you reject at $\alpha = .05$.

