Jihye Kim (Sarah)
Educational Policy Study:
Research, Measurement, and Statistics
Georgia State University

An Excel manual for Item Response Theory.

1. Item Characteristic Curve
2. Maximum Likelihood of θ
3. Item and Test Information Function

1. How to draw Item Characteristic Curve with using Excel

In item response theory, the relationship between examinee's item performance and the set of traits can be described by item characteristic curve (ICC). There are several ways to draw ICC, but I will show you how to draw it using Excel.

In figure 1, table 1 shows the set of item parameters for 4 items. The value of the parameter *b* for item 3 is in cell E6. Alphabets in the first row and the numbers in the first column are used for naming cells in Excel.

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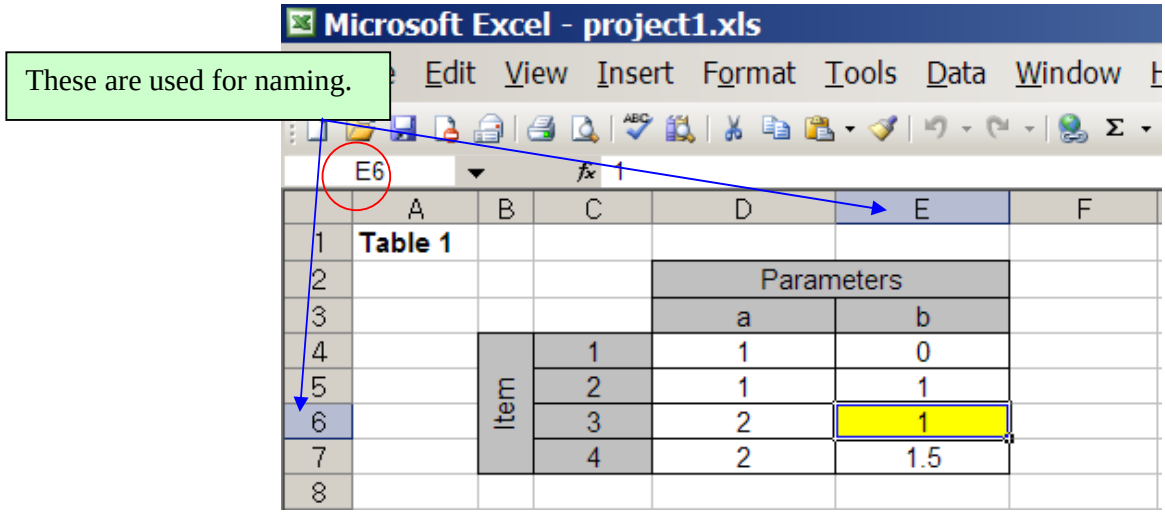


Figure 1

In order to draw ICC, we should first calculate $P(\theta)$. The formula for $P(\theta)$ is as follows:

$$P(\theta) = \frac{\exp\left(1.7 * a * (\theta - b)\right)}{1 + \exp\left(1.7 * a * (\theta - b)\right)}$$

Above formula can be easily implemented in Excel. For example, if we want to calculate $P(-3)$ for item 1, we need the values of parameters a and b for two-parameter model. The values of a, b, and θ for calculating P(-3) are in cells D4, E4, and D11, respectively. In Excel programming, it is always recommended to use cell reference (or name) instead of embedding values directly in the formula. Therefore, writing a formula in cell D12 as in the following way is highly discouraged.

```
= EXP(1.7*1*(-3-0))/(1+EXP(1.7*1*(-3-0)))
```

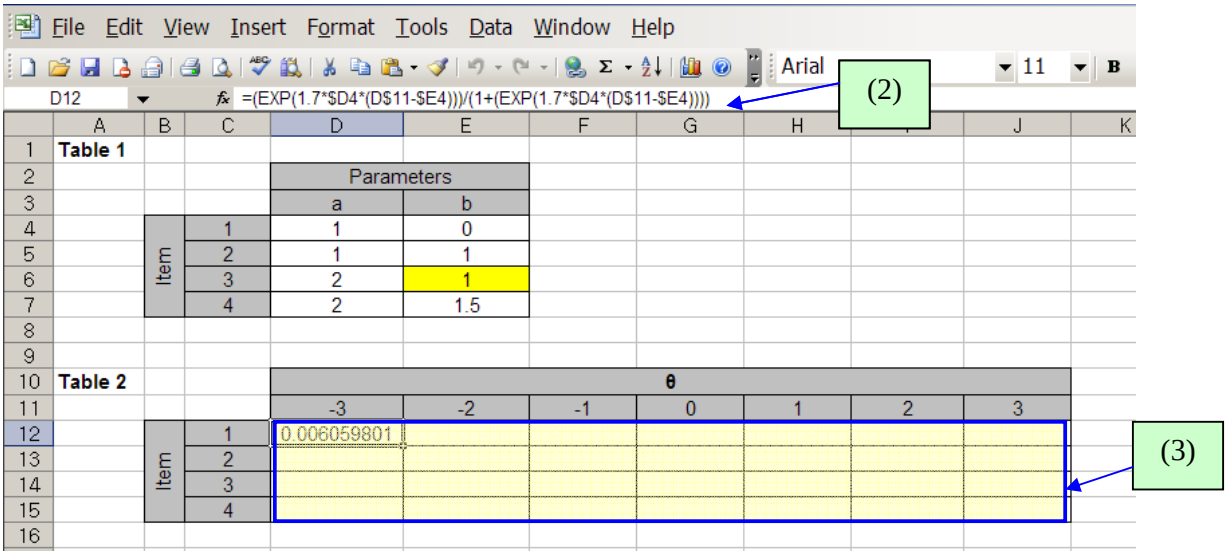
Instead, this formula should be written as

```
= EXP(1.7*D4*(D11-E4)))/(1+(EXP(1.7*D4*(D11-E4)))        (1)
```

using cell references as shown in figure 2. If you look closely the formula bar, you will find there is a slight difference between (1) and (2). Using $ in the cell reference will make the reference as an absolute reference. This means when you copy the cell D12 and paste in other

cell, the cell reference will not be changed. In Excel, if the formula is copied to a new location, it is adjusted to reflect new column or row locations. However, in this case, we do not want all of a formula to adjust as it is copied, so we need to use *absolute referencing*. This means that the formula is referencing a fixed location, is absolute, and does not adjust as it is copied.

Once we finished with typing a correct formula in cell D12, we copy the cell D12 by pressing Ctrl-C. Next, highlight cells D12:J15 ((3) in figure 2), and then paste it by pressing Ctrl-V. With this method, we can fill each cell in the table 2 from D12 to J15.



	A	B	C	D	E	F	G	H	J	K
1	**Table 1**									
2				Parameters						
3				a	b					
4		Item	1	1	0					
5			2	1	1					
6			3	2	1					
7			4	2	1.5					
8										
9										
10	**Table 2**			θ						
11				-3	-2	-1	0	1	2	3
12		Item	1	0.006059801						
13			2							
14			3							
15			4							
16										

Figure 2

Figure 3 shows the value of $P(\theta)$ for each item with corresponding θ.

	A	B	C	D	E	F	G	H	J	K
9										
10	**Table 2**			θ						
11				-3	-2	-1	0	1	2	3
12		Item	1	0.006059801	0.032295465	0.1544653	0.5	0.8455347	0.9677045	0.9939402
13			2	0.001112536	0.006059801	0.0322955	0.1544653	0.5	0.8455347	0.9677045
14			3	1.24049E-06	3.71689E-05	0.0011125	0.0322955	0.5	0.9677045	0.9988875
15			4	2.26618E-07	6.79036E-06	0.0002034	0.0060598	0.1544653	0.8455347	0.9939402
16										

Figure 3

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Once all probabilities for four items are calculated, we can draw ICC. In order to draw the graph, click Insert and then Chart. (See figure 4)

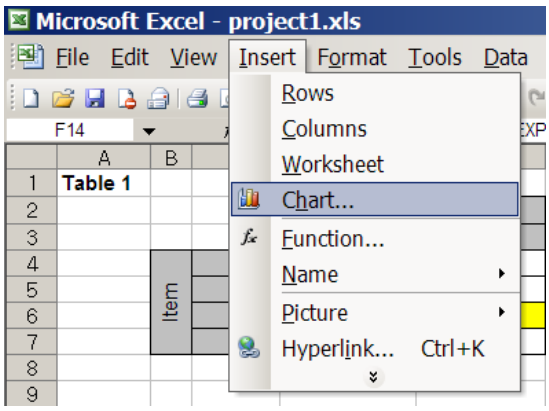
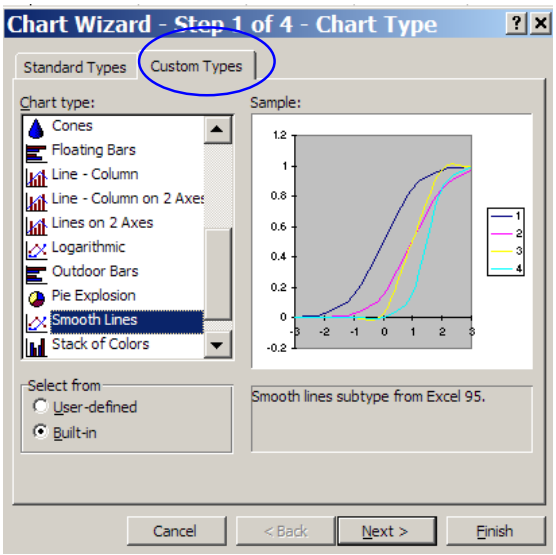


Figure 4

In the pop-up window as shown in figure 5, choose 'Custom types' tap, and then click 'smooth lines.' Click 'Next.'



Another pop up widow will show up. Here we have to select data range, which is C11:J15. The values in D11:J11 will be served as values for X-axis, and the values in C12:C15 will be served as labels of the graphs. Once you selected data range, click 'Next.' This data range selection can be done before the step shown in figure 4. We will show this later in section 3.

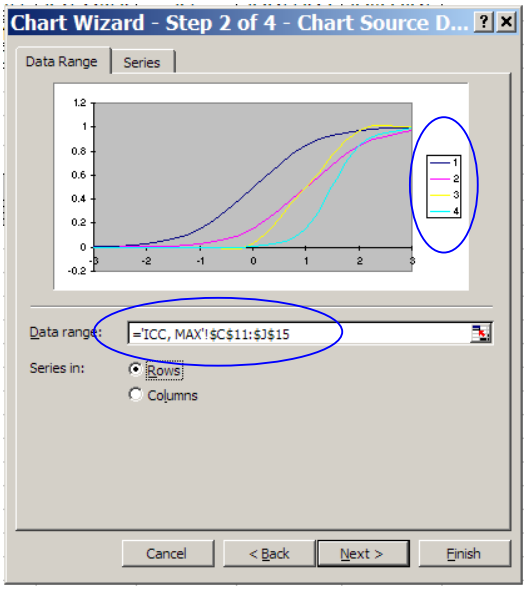


Figure 6

At this window, we need type the chart title, Category (X) axis and Value (Y) axis. Put the cursor in each white box on the left side, which is marked with thick circled line in the window, then type. And click 'Next'.

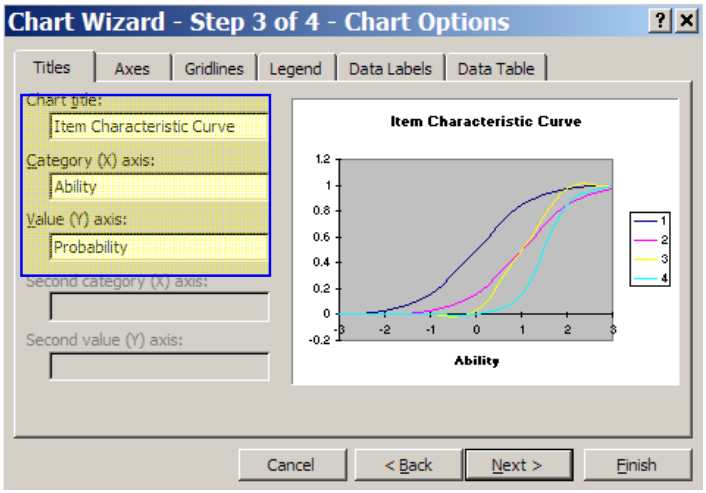


Figure 7

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Click ‘Finish’.

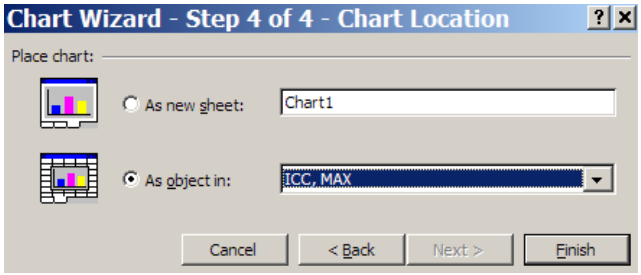

Figure 8

Then we can get the Item Characteristic Curve as follows. In figure 9, we can see the Y-axis, which is probability, needs adjustments so that Y-axis begins from zero and ends at one.

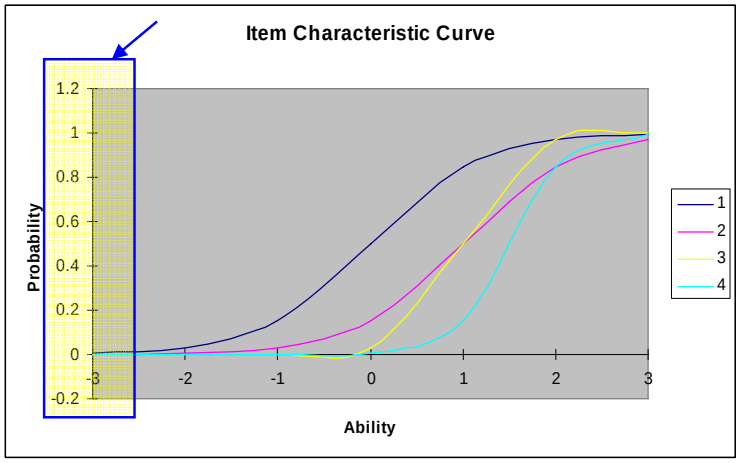

Figure 9

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Now, we are going to adjust the scale of Y-axis in the graph. First, put the cursor at the Y-axis in the graph, and then right-click. Then the small pop-up will show up; here, select 'Format Axis',

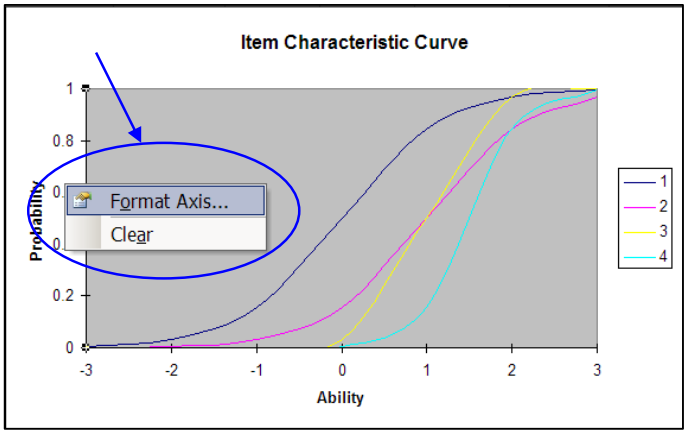


Figure 10

Then this popup window comes out. In this popup, select 'Scale' tap and change 'Minimum' to 0, 'Maximum' to 1. Click 'OK'.

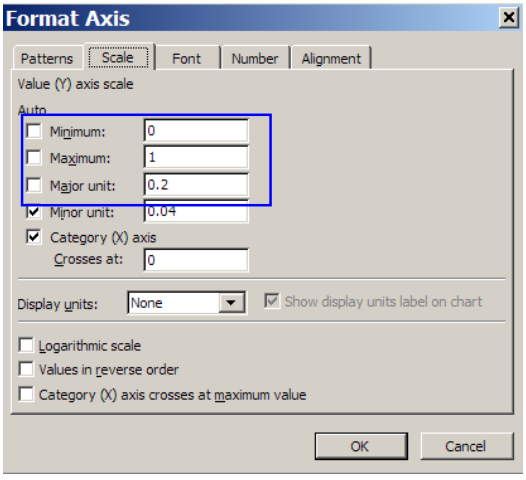


Figure 11

Now, we can see the adjusted Y-axis as below thick squared line.

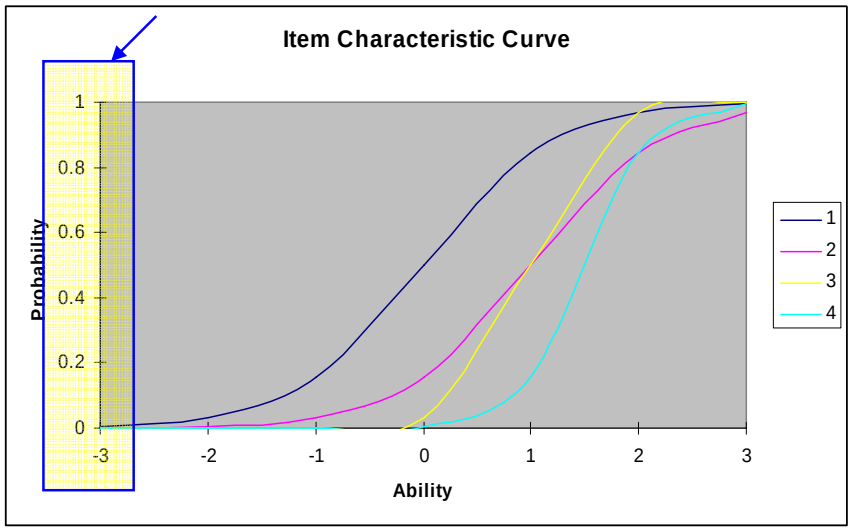
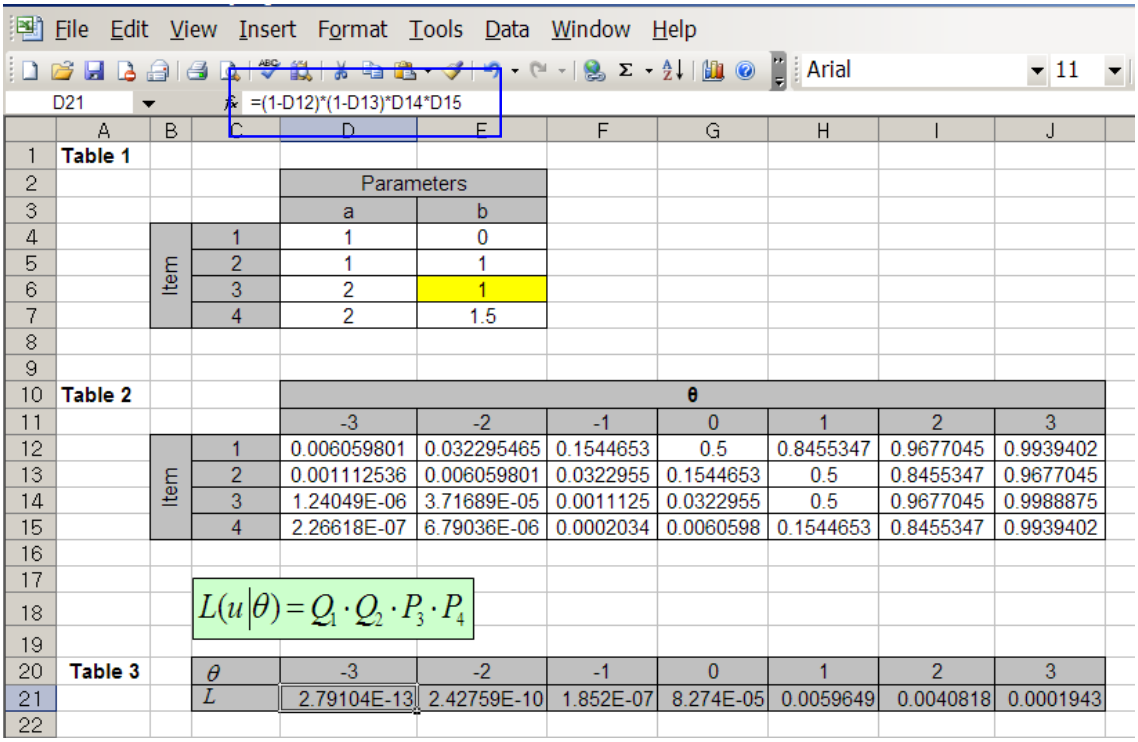


Figure 12

2. How to find Maximum value using specific function in excel.

In this section, I will show you how to find θ that maximizes the likelihood function, $L(u|\theta)$. To start, we need to set up the formulas for the likelihood function. Table 3 in figure 13 shows values of $L(u|\theta)$ corresponding to different values of θ. As shown in formula bar, table 3 is dependent on the values in table 2. Also notice that values in table 2 are calculated using parameters, *a* and *b*, from table 1 and θ*'s* from the row D11:J11. That means, values in table 3 are dependent on the θ values in row D11:J11.



	A	B	C	D	E	F	G	H	I	J
1	**Table 1**									
2				Parameters						
3				a	b					
4		Item	1	1	0					
5			2	1	1					
6			3	2	1					
7			4	2	1.5					
8										
9										
10	**Table 2**			θ						
11				-3	-2	-1	0	1	2	3
12		Item	1	0.006059801	0.032295465	0.1544653	0.5	0.8455347	0.9677045	0.9939402
13			2	0.001112536	0.006059801	0.0322955	0.1544653	0.5	0.8455347	0.9677045
14			3	1.24049E-06	3.71689E-05	0.0011125	0.0322955	0.5	0.9677045	0.9988875
15			4	2.26618E-07	6.79036E-06	0.0002034	0.0060598	0.1544653	0.8455347	0.9939402
16										
17										
18			$L(u\|\theta)=Q_1\cdot Q_2\cdot P_3\cdot P_4$							
19										
20	**Table 3**		θ	-3	-2	-1	0	1	2	3
21			L	2.79104E-13	2.42759E-10	1.852E-07	8.274E-05	0.0059649	0.0040818	0.0001943
22										

Figure 13

Once table 3 is completed, it is ready to use Solver to find θ that maximize the likelihood function.[1] Click 'Solver' under 'Tools' menu. In Solver Parameters pop-up window, select the cell that we want to maximize in 'Set Target Cell.' Here, any cell in table 3 (D21:J21) will work as target cell, because we are trying to maximize the likelihood function. Let us choose cell D21. Next, we should specify what we want to do with the target cell. Since we want to maximize the likelihood function, click 'Max' radio button. Next, we have to specify which cell(s) we want to manipulate to maximize the target cell. In this case, we want to manipulate θ to get maximum value of likelihood function. Therefore, we have to choose the cell D11 as θ, because the target cell D21 is linked with the cell D11. Choose D11

[1] Solver is not installed as a default package. If you do not see Solver under Tools menu, you may have to install it.

for 'By Changing Cells.' Since there is no other constraint (e.g., θ must be positive.), we are ready to find out optimum θ value by clicking 'Solve.'

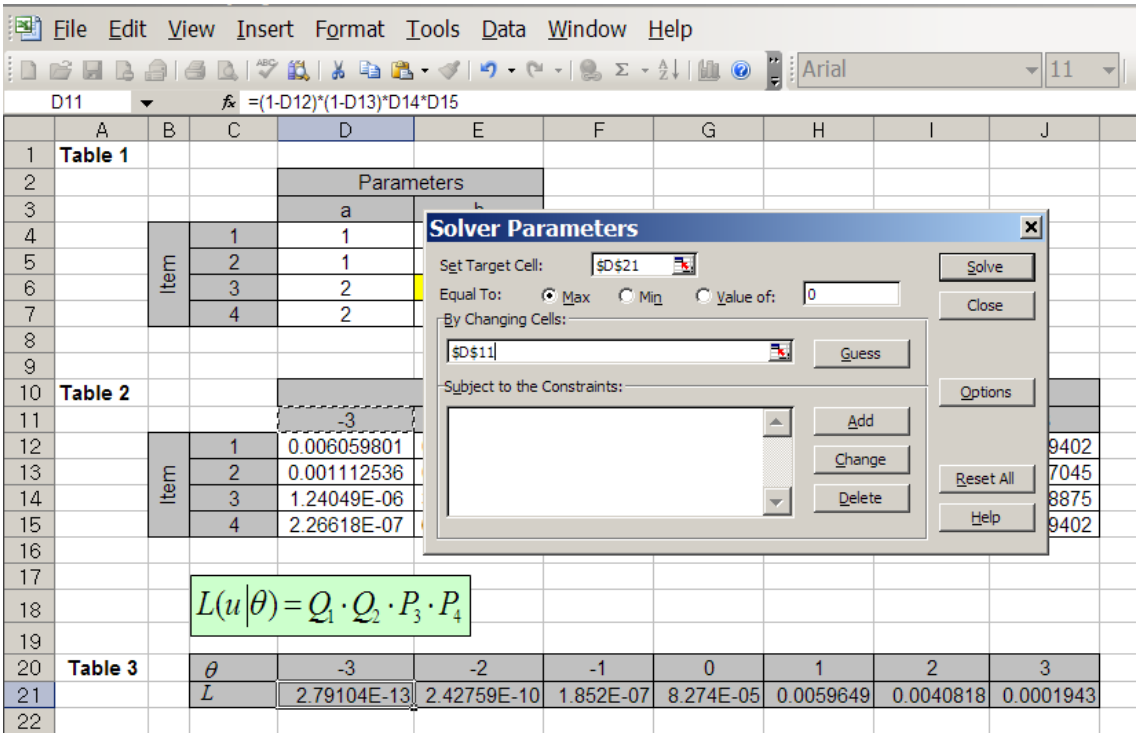


Figure 14

Once you click solve, Solver says it found a solution. However, there is no change in θ value, cell D11. This is one of drawbacks that Solver has. Solver is essentially a trial-and-error procedure, and it tries different values for θ many times (maximum 9,999 times). If it cannot find a converged solution, it will show the initial value for θ or some not-correct solution. If this happens, changing the initial value may be necessary. Since we can tell that maximum of likelihood function happens between 0 and 2 from table 3, we can try 1 as an initial starting value.

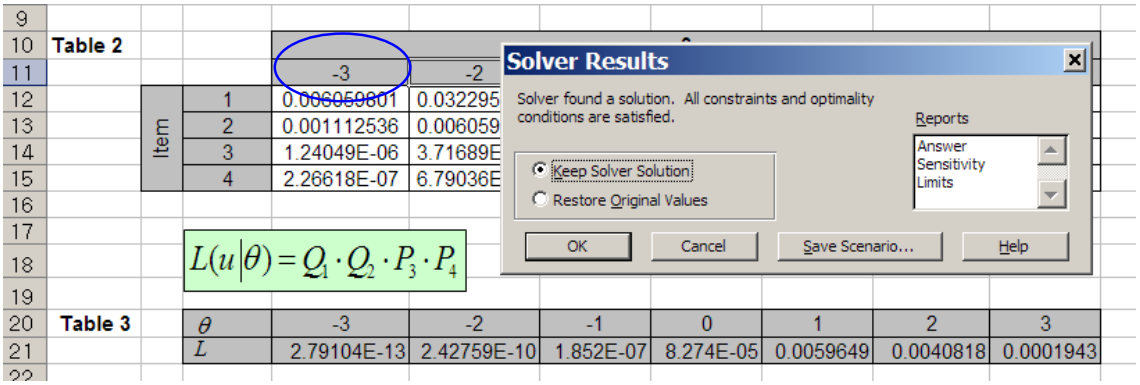


Figure 15

Once 1 is used as a starting value, Solver found a correct solution, which is 1.399 (figure 16).

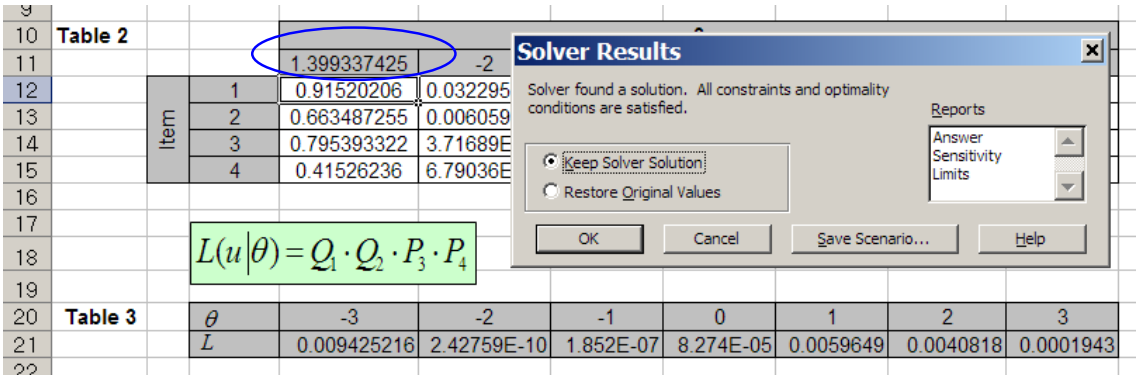


Figure 16

3. Item and Test Information Function.

The formula of item information function, $I_i(\theta)$, is shown in figure 17. How to calculate $I_i(\theta)$ in Excel is also shown in the formula bar in figure 17. The calculation procedure is very similar to that of $P(\theta)$, as we saw before.

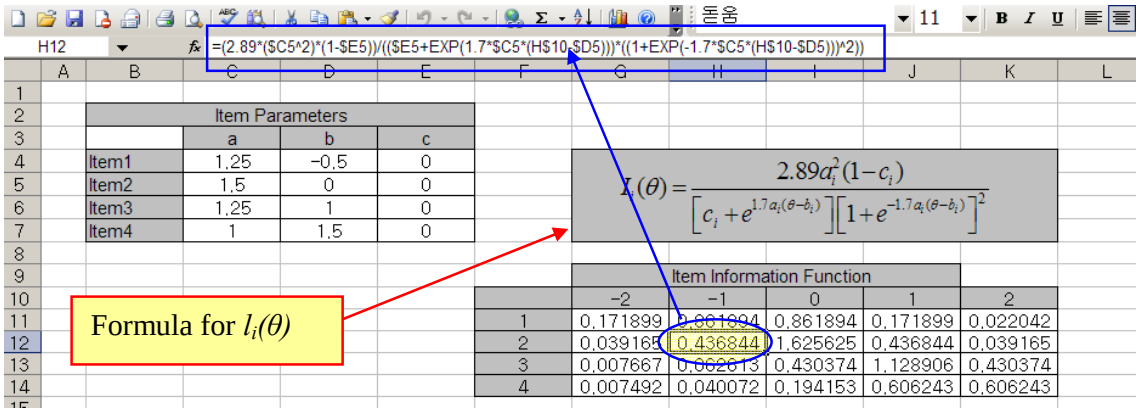


Figure 17

With these item information functions, we plot the graph. The procedure is similar as in the case of ICC. First select all cells that we want to draw the graph.

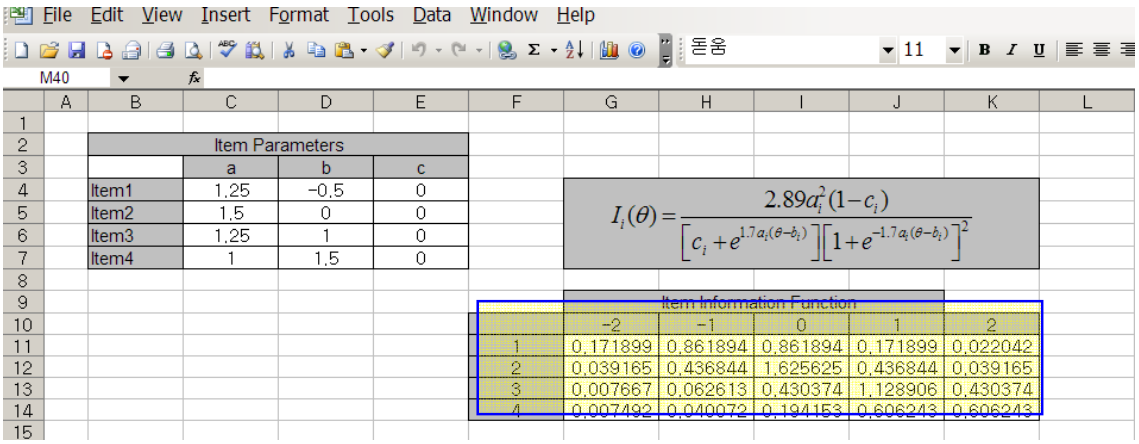


Figure 18

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Click Chart under Insert menu.

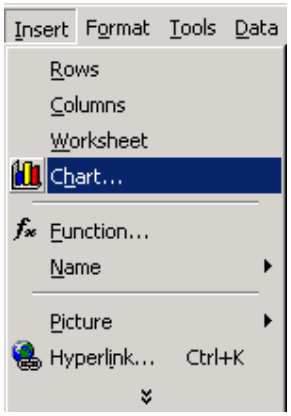


Figure 19

Select ‘Smooth Lines’ in the popup windows, and

Click the ‘Next’.

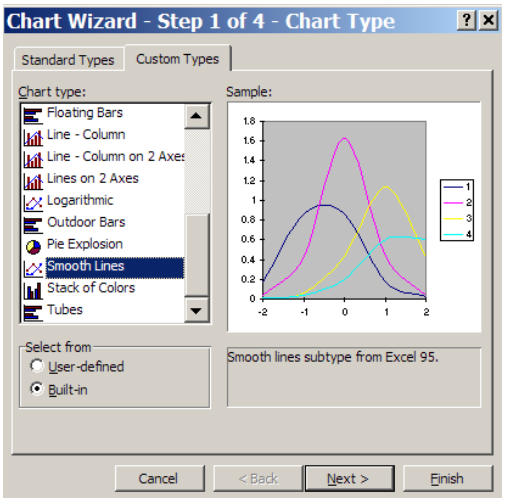


Figure 20

Click ‘Next’.

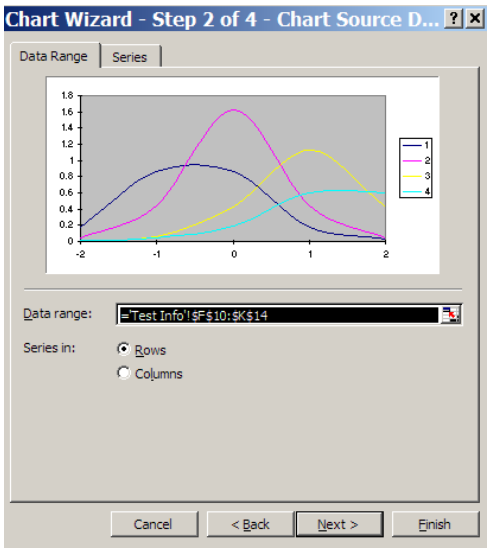


Figure 21

Type appropriate names in ‘Chart title’, ‘Category (X) axis, and ‘Value (Y) axis. Then Click ‘Next’.

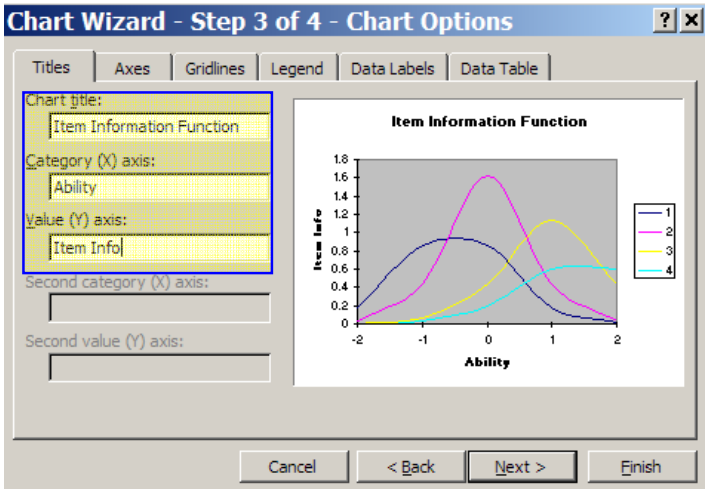


Figure 23

Click ‘Finish’

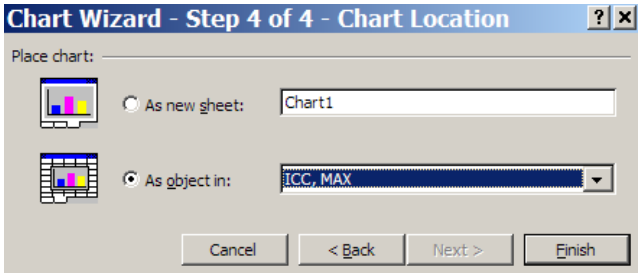


Figure 24

Now, we see the graph of Item Information Function plot at figure 25.

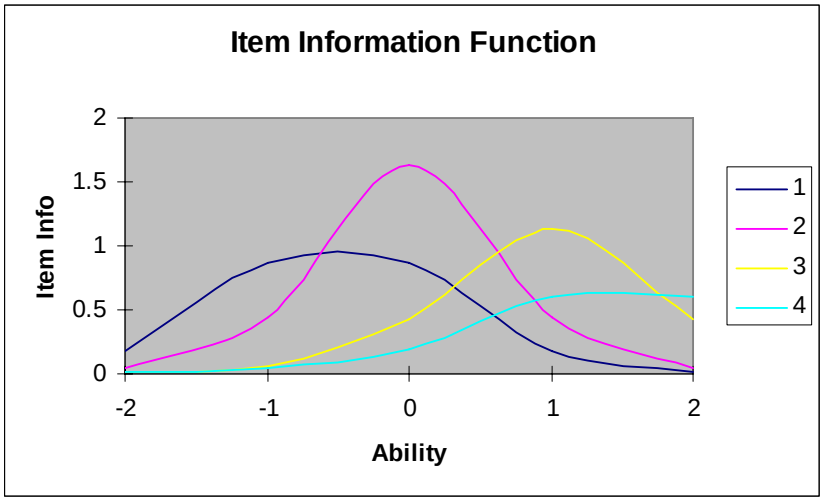
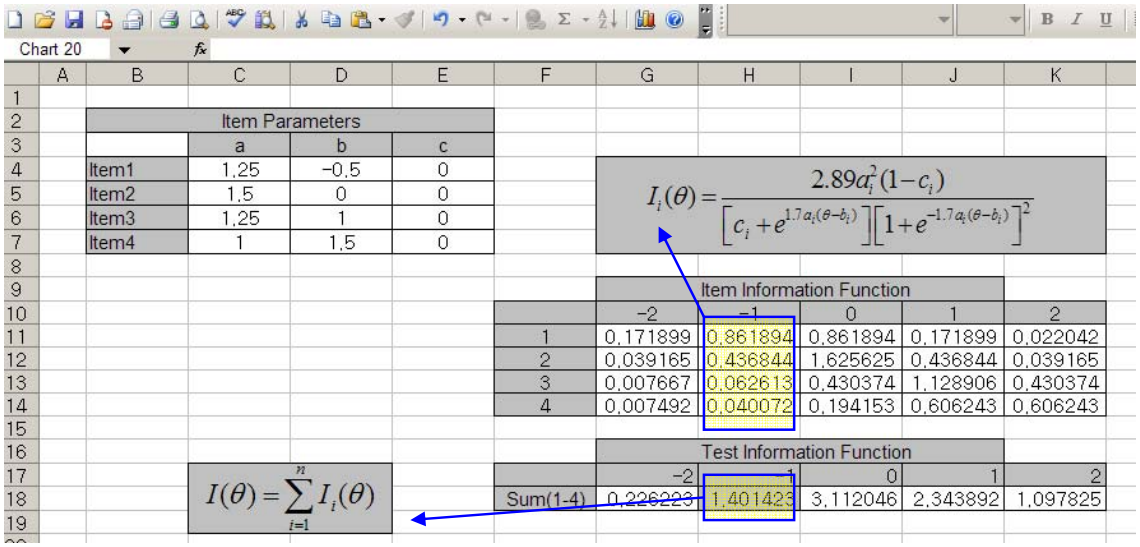


Figure 25

When we calculate the test information of 4 items, we add all $I_i(\theta)$ at each ability level θ as follows.

$$I(\theta) = \sum_{i=1}^{4} I_i(\theta) = 0.861894 + 0.426844 + 0.062613 + 0.040072 = 1.401423$$

In Excel, instead of using formula `=H11+H12+H13+H14`, it can be simplified to the formula `=sum(H11:H14)`.



	Item Parameters		
	a	b	c
Item1	1.25	−0.5	0
Item2	1.5	0	0
Item3	1.25	1	0
Item4	1	1.5	0

$$I_i(\theta) = \frac{2.89a_i^2(1-c_i)}{\left[c_i + e^{1.7a_i(\theta-b_i)}\right]\left[1+e^{-1.7a_i(\theta-b_i)}\right]^2}$$

	Item Information Function				
	−2	−1	0	1	2
1	0.171899	0.861894	0.861894	0.171899	0.022042
2	0.039165	0.436844	1.625625	0.436844	0.039165
3	0.007667	0.062613	0.430374	1.128906	0.430374
4	0.007492	0.040072	0.194153	0.606243	0.606243

	Test Information Function				
	−2	−1	0	1	2
Sum(1-4)	0.226223	1.401423	3.112046	2.343892	1.097825

$$I(\theta) = \sum_{i=1}^{n} I_i(\theta)$$

Figure 26

From the data in figure 26, we can draw Test Information Function Plot as below. It is similar procedure with the case of ICC and that of Item Information Function Plot.

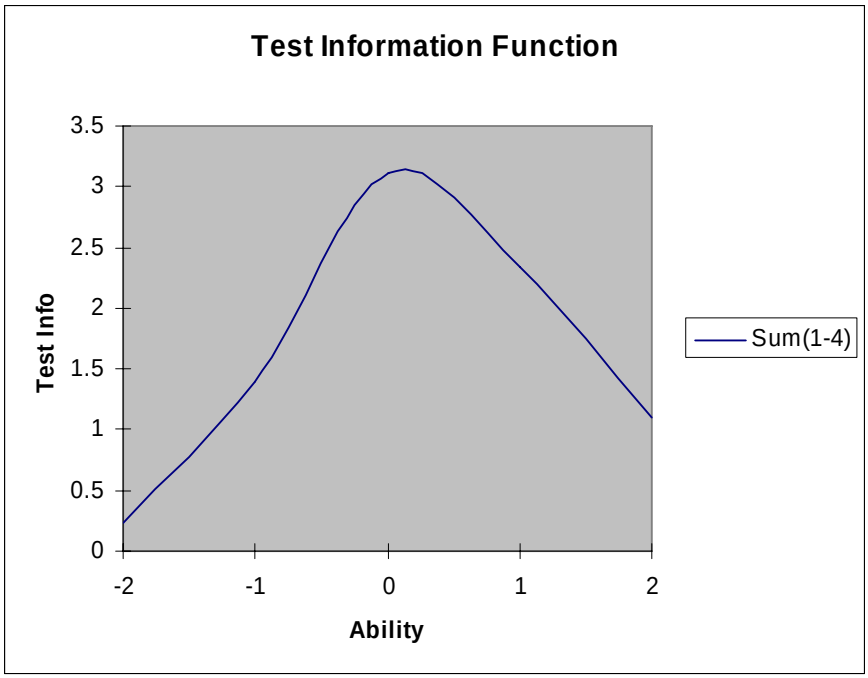


Figure 27